BASEMENT

050-79

WORKING PAPER
ALFRED P. SLOAN SCHOOL OF MANAGEMENT

MARKETING MULTIPLIER AND MARKETING STRATEGY SIMPLIFIED DYNAMIC DECISION RULES*

Hermann Simon**

Working Paper 1050-79 March 1979

MASSACHUSETTS
INSTITUTE OF TECHNOLOGY
50 MEMORIAL DRIVE
CAMBRIDGE, MASSACHUSETTS 02139

MARKETING MULTIPLIER AND MARKETING STRATEGY SIMPLIFIED DYNAMIC DECISION RULES*

Hermann Simon**

Working Paper 1050-79 March 1979

* The author is indebted to Manu Kalwani and Alvin J. Silk for helpful comments.

** Assistant Professor of Management Science,
University of Bonn, and
Visiting Fellow, Sloan School of Management,
Massachusetts Institute of Technology

ABSTRACT

For a wide class of empirically tested dynamic marketing response models a heuristic method for the determination of dynamically optimal price and promotion levels is being developed. The future effects of present marketing actions are measured by a marketing multiplier which is partially based on managerial estimates. Very simple dynamic optimality conditions for both single marketing variables and the marketing mix are formulated. The application of the method to a number of empirical models yields interesting insights into realistic magnitudes of the dynamic impact.

736280

INTRODUCTION

Within the last few years a large body of empirical work on dynamic marketing response models has emerged. In these models, the dependent variable sales or market share is usually considered as a function of the lagged dependent variable and of one or several marketing instruments. A very genereral form of these dynamic response functions can be written as

$$q_t = a_t + \lambda\, q_{t-1} + f_t(x_{1,t},\ldots,x_{n,t}) \qquad (1)$$

where
- q_t sales in period t (units or market share)
- $f_t(.)$ marketing response function in period t
- $x_{j,t}$ value of marketing variable j in period t (units or share)
- a_t absolute term (parameter)
- λ_t carryover-coefficient (parameter)

The variables in (1) can be either in natural or in logarithmic dimension so that the function comprehends both the linear and the multiplicative sales model. Moreover, the parameters and/or the marketing responses can be either constant or time-varying.

Reviews of a great number of empirically tested models of type (1) can be found in [10, 11, 28, 39]. Table 1 provides a synopsis of the different versions encountered in the marketing literature. These studies comprehend more than 200 products or product-market-combinations.

INSERT TABLE 1 SOMEWHERE HERE

Due to the wide coverage in the literature it doesn't seem necessary to repeat the rationale which underlies the different versions of model (1). We shall focus on deriving a simple dynamic optimization heuristic which can be applied to almost all of these versions.

The numerous optimization approaches in the literature are almost exclusively limited to advertising and the linear carryover-function. Optimality conditions of this type are, for instance, given in [3, 15, 17, 19, 35, 48]; numerical solutions can be found in [8, 27, 47]. Another group of approaches use modern control theory in order to derive dynamic optimality conditions for advertising. Schmalensee [32] gives very general conditions of this type but does not provide any numerical solution. Most of the control theoretic approaches are limited to specific advertising models (in particular to the models of Vidale-Wolfe [46] and Nerlove-Arrow [25]) which will not be investigated in this paper, [4, 13, 14, 33, 43, 44] are of this type). An exception is the pricing model of Spremann [40, 41] a version of which can be compared with function (1). To date, the practical relevance of the control theoretic models has remained very limited.

We are not aware of any work in which a unified optimization approach for all versions of model (1) and different marketing instruments is provided. The present paper is structured as follows. In the next section we define a simple measure of the cumulative effects of a marketing action. This measure is subsequently used to formulate optimality conditions. Finally a simplified procedure which takes advantage of these conditions in order to obtain numerical solutions is proposed and a number of applications is discussed.

CUMULATIVE MARKETING EFFECTS

The applicability of the following derivations is limited to models in which the carryover-effect and the sales response to the marketing variable on which a decision is to be made are separable. This separability condition holds for all models in table 1 with the exception of [24, 45, 50].

Measuring the short-run sales effect of a certain marketing activity $x_{j,t}$ by

means of the partial derivative $\partial q_t / \partial x_{j,t}$, we obtain the total cumulative sales effect (over time) attributable to this short-run response as

$$\Delta Q_t = \sum_{\tau=0}^{\infty} \frac{\partial q_t}{\partial x_{j,t}} \frac{\partial q_{t+\tau}}{\partial q_t} = \frac{\partial q_t}{\partial x_{j,t}} \sum_{\tau=0}^{\infty} \frac{\partial q_{t+\tau}}{\partial q_t} \qquad (2)$$

The sum term on the right hand side of (2) gives the total cumulative sales effect of a marketing action as a multiple of the action's short-run effect. Therefore, it seems reasonable to denote this sum term as marketing multiplier.

In the case of a linear carryover-function the multiplier for which we write m_t is simply obtained as

$$m_t = 1 + \lambda_t + \lambda_t \lambda_{t+1} + \lambda_t \lambda_{t+1} \lambda_{t+2} + \ldots\ldots \qquad (3)$$

and if the carryover-coefficient λ is constant over time and $0 < \lambda < 1$

$$m_t = 1/(1 - \lambda) \qquad (4)$$

which is the expression first derived by Palda [26]. Kotler [16] denoted (4) as "long run marketing expenditure multiplier" and more recently - obviously unaware of Kotler's denomination - Dhalla [11] used the label "long-term marketing multiplier" for (4).

In the linear case, m_t is independent from the future marketing activities, whereas it depends on those activities in the multiplicative form of model (1). The marketing multipliers of all models under consideration are given in table 2.

INSERT TABLE 2 HERE

By means of the marketing multiplier m_t and the short-run elasticity a long-run marketing elasticity $E_{j,t}$ can be defined in the following way

$$E_{j,t} = m_t \cdot e_{j,t}$$

where $e_{j,t} = \partial q_t / \partial x_{j,t} \cdot x_{j,t}/q_t$ is the usual short-run elasticity.

$E_{j,t}$ gives the cumulative sales effect which is induced by a 1%-change in marketing variable j as a percentage of current sales. Though, to date, little observed in the literature, this elasticity seems to have highly interesting empirical properties. Comparing the respective price elasticities of 12 detergents and 21 pharmaceuticals the author obtained the following amazing results [39]

	short-run elasticity (mean)	marketing multiplier (mean)	long-run elasticity
Detergents	2.37	1.75	4.15
Pharmaceuticals	.76	3.64	2.77

Though the short-run price elasticities are highly different, the long-run elasticities of both product groups are rather close together (not significantly different at the 5%-level) due to the differences in the carryover-patterns.

It should be observed that both m_t and $E_{j,t}$ are sales (quantity) related measures of long-run marketing effects. In order to make optimal marketing decisions, however, a value-related measure of the long-run effects is required. We obtain this measure by weighing each period $(t+\tau)$'s term in m_t by the contribution margin $d'_{t+\tau}$ and the discount factor $(1 + r)^{-\tau}$. The resulting value-adjusted marketing multiplier is

$$m'_t = d'_t + \sum_{\tau=1}^{\infty} d'_{t+\tau} \frac{\partial q_{t+\tau}}{\partial q_t} (1+r)^{-\tau} \qquad (5)$$

For notational purposes we also need the respective term which excludes the contribution of period t, we write for this term m''_t

$$m''_t = m'_t - d'_t = \sum_{\tau=1}^{\infty} d'_{t+\tau} \frac{\partial q_{t+\tau}}{\partial q_t} (1+t)^{-\tau} \qquad (6)$$

Thus, m'_t denotes the present value of the cumulative sales effect produced by a marginal change in q_t and m''_t is the respective value of all future periods (excluding t).

GENERAL AND SPECIFIC OPTIMALITY CONDITIONS

The objective function to be maximized in period t is the sum of all discounted future profits

$$\Pi_t = \sum_{\tau=0}^{\infty} [p_{t+\tau}\, q_{t+\tau} - C_{t+\tau}(q_{t+\tau}) - \sum_{j=2}^{n} x_{j,t+\tau}\,](1+r)^{-\tau} \qquad (7)$$

where $p_{t+\tau} = x_{1,t+\tau}$ shall denote the price in period $t+\tau$, $C_{t+\tau}(q_{t+\tau})$ is the cost function, and $q_{t+\tau}$ is determined according to the dynamic response model.

In order to maximize (7) a hypothesis on the presumable reaction of competitors to the firm's marketing actions is required. We assume that competitive reactions are limited to the same marketing instrument, the so-called "simple direct competitive reaction case" [18, p.23], and to the same period so that the reaction function can be written as $\bar{x}_{j,t} = g(x_{j,t})$ where $\bar{x}_{j,t}$ denotes a unidimensional measure of the marketing efforts of all competitors in instrument j, e.g. the sum of advertising expenditures, a market share weighted average price etc. If this reaction pattern holds $\bar{x}_{j,t}$ can be replaced by $g(x_{j,t})$ and the necessary conditions for the maximum of (7) can be derived.

Since the price p_t and the non-price marketing variables $x_{2,t},\ldots,x_{n,t}$ enter (7) in different ways the optimality conditions for both types of variables are derived separately.

$$\frac{\partial \Pi_t}{\partial p_t} = q_t + (p_t - C_t')\frac{\partial q_t}{\partial p_t} + \sum_{\tau=1}^{\infty} d_{t+\tau}' \frac{\partial q_{t+\tau}}{\partial p_t}\,(1+r)^{-\tau} = 0 \qquad (8)$$

$$\frac{\partial \Pi_t}{\partial x_{j,t}} = -1 + (p_t - C_t')\frac{\partial q_t}{\partial x_{j,t}} + \sum_{\tau=1}^{\infty} d_{t+\tau}' \frac{\partial q_{t+\tau}}{\partial x_{j,t}}\,(1+r)^{-\tau} = 0 \qquad (9)$$

where C_t' denotes marginal cost and $d_{t+\tau}' = (p_{t+\tau} - C_{t+\tau}')$ is the marginal contribution in period $t+\tau$.

We readily recognize from (2) and (6) that the sum terms in (8) and (9) are

equal to the product of the value adjusted multiplier m''_t (excluding period t) and the short-run response $\partial q_t/\partial x_{j,t}$, j=1,...,n, so that (8) and (9) can be rewritten as

$$q_t + \frac{\partial q_t}{\partial p_t}(p_t - C'_t + m''_t) = 0 \qquad (10)$$

$$-1 + \frac{\partial q_t}{\partial x_{jt}}(p_t - C'_t + m''_t) = 0 \qquad (11)$$

with m''_t according to (6). Multiplying (10) and (11) by p_t/q_t and $x_{j,t}/q_t$ respectively and solving for the optimal values p^*_t and $x^*_{j,t}$ leads to

$$p^*_t = \frac{e_{p,t}}{1 + e_{p,t}}(C'_t - m''_t) \qquad (12)$$

$$x^*_{j,t} = e_{j,t}\, q_t(p_t - C'_t + m''_t) \qquad (13)$$

where

$$e_{p,t} = \partial q_t/\partial p_t \cdot p_t/q_t \qquad (14a)$$

$$e_{j,t} = \partial q_t/\partial x_{j,t} \cdot x_{j,t}/q_t \qquad j = 2,...,n \qquad (14b)$$

are the short-run price and promotion elasticities respectively.

For some function types, conditions (12) and (13) do not allow for a direct computation of the optimal values since the right hand sides still depend on the left hand variable. The two conditions reveal, however, very clearly the structure of the dynamically optimal marketing decisions and their relation to the respective static values. The latter are simply obtained by setting $m''_t = 0$.

Thus, the basic difference between dynamic and static optimality conditions is that, in the dynamic case, marginal cost C'_t is diminished by the present value m''_t of future profits induced by a change in the marketing variable under consideration. Economically, the term $(C'_t - m''_t)$ can be interpreted as "dynamic marginal cost".

The relation between optimal dynamic and optimal static values is determined by the sign and the magnitude of m''_t; m''_t is positive if $\lambda_{t+\tau}$ and $d'_{t+\tau}$ are positive for all τ. It can even then be positive if some $\lambda_{t+\tau}$ and/or $d'_{t+\lambda}$ are negative. The author is only aware of three cases in which λ turned out to be negative and significantly different from zero (1 product in [29] and 2 products in [38]). Since the marginal contribution d' is equally unlikely to be negative in the long-run, m''_t can typically be expected to be positive.

If this holds we can infer the following general findings from (12) and (13):

(1) The optimal dynamic price is lower than the optimal static price.

(2) The optimal dynamic promotion expenditure is greater than the optimal static expenditure.

(3) In both cases the differences between optimal dynamic and optimal static values are, ceteris paribus, the greater
   - the greater the future contribution margins $d'_{t+\tau}$ are,
   - the smaller the discount rate r is,
   - the greater the carryover-coefficients $\lambda_{t+\tau}$, $\tau = 0,\ldots,\infty$ are

Relating conditions (12) and (13) we obtain a dynamic version of the Dorfman-Steiner-Theorem which - as is well known - defines the optimality conditions for the marketing mix. We first insert (12) into (13) and form the relation

$$\frac{p^*_t}{x^*_{j,t}} = \frac{e_{p,t}/(1+e_{p,t}) \cdot [C'_t - m''_t]}{e_{j,t}\, q_t \{e_{p,t}/(1+e_{p,t})\, [C'_t - m''_t] - C'_t + m''_t\}} \tag{15}$$

This expression can be considerably simplified and written in the usual form of the Dorfman-Steiner-Theorem

$$\frac{p^*_t\, q^*_t}{x^*_{j,t}} = - \frac{e_{p,t}\, m''_t}{e_{j,t}\, m''_t} = - \frac{e_{p,t}}{e_{j,t}} \tag{16}$$

The intermediate term in (16) is formally equivalent to Schmalensee's result [32] which is, however, based on the assumption of an equilibrium situation and the derivation of which required a more complex control theoretic approach. The conditions derived by Jacquemin [14] and Bensoussan et al. [5] are equivalent in substance but different in form since they apply to a model of the Nerlove-Arrow-Type. Under the aspect of applicability all of these models have the additional disadvantage of being continuous with respect to time.

The most important result in (16) is that - in the case of a separable dynamic sales response function of type (1) - the optimal sales-promotion-ratio is the same under dynamic and under static conditions. Note,however, that the magnitudes of optimal dynamic and optimal static values of the marketing variables are almost always different (as outlined above).

This result which is more specific than the aforementioned conditions [5, 14, 32] is conclusive since - due to the separability of carryover-effect and short-run response - price and non-price variables produce exactly the same carryover-pattern, i.e. m''_t appears both in the denominator and the numerator so that the simplification which leads to the right hand side of (16) can be made. Note that (16) does not imply that the sales-promotion-ratio should be constant over time. Rather the contrary has to be expected since both price and promotion elasticities are highly likely to change over time [2, 27, 38, 49].

Our relatively simple derivations call for a comparison with more complex (in particular control theoretic) approaches which serve the same purpose. Most of the aforementioned control theoretic models do, however, not allow for a direct comparison since they are based on functions different from type (1) [5, 13, 14, 33, 43, 44]. One exception is the model of Spremann

[40, 41] in a version of which the continuous analogon of function (7) is maximized. In control theory, the maximization of the original objective function is achieved by maximizing an intermediate function, the so-called Hamiltonian

$$H = [p(t) - C(t)]\, q(t)\, e^{-\rho t} + v^*(t)\, q(t) \tag{17}$$

where (in addition to our usual notation)

e base of natural logarithms

ρ discount rate for continuous time

$v^*(t)$ adjoint variable

The adjoint variable evaluates the state of the system at any point in time and is determined by the differential equation

$$\frac{\partial v(t)}{\partial t} = - \{e^{-\rho t} [p^*(t) - C'(t)] + v^*(t) \} \frac{\partial q(t)}{\partial s(t)} \tag{18}$$

and a terminal boundary condition

$$v(T) = 0 \tag{19}$$

where s(t) is a state variable (however defined) and T denotes the end of the planning horizon. An extended economic interpretation of the adjoint system can be found in [5]. Maximizing the Hamiltonian (17) with respect to p(t) leads to the condition [40, 41]

$$p^*(t) = \frac{e_p(t)}{1+e_p(t)} [C'(t) - v^*(t)/e^{-\rho t}] \tag{20}$$

subject to (18) and (19). Comparing (20) with (12) we readily recognize that the value-adjusted marketing multiplier m_t'' in (12) is equivalent - not necessarily equal since $p^*(t) \neq p_t^*$ in almost all t - to the discounted adjoint variable in (20). Analoguous conditions can be derived for non-price marketing variables.

This equivalence is of utmost importance since it shows that conditions (12) and (13) contain the same economic information as a control theoretic model. These conditions have, however, some considerable advantages over their control theoretic equivalents. They apply to discrete, empirically well-founded models, can easily be interpreted economically and explained to managers, and are more suitable for computational purposes.

For most of the models encountered in the literature the general optimality conditions given in (12) and (13) take on very simple forms.

It should be mentioned that many of the empirically tested advertising response functions are linear so that no finite positive optimal advertising level exists [e.g. 2, 9, 26]. Optimality conditions for specific models which have a finite optimum are given subsequently.

(1) Linear, time-invariant carryover, logarithmic advertising response [36, some versions in 26]

$$q_t = a + \lambda q_{t-1} + b \cdot \ln A_t \tag{21}$$

Optimal advertising:

$$A_t^* = b \cdot (p_t - C_t' + m_t'') \tag{22}$$

and, in the particular case, where the future contribution margins are assumed to be constant, $d'_{t+\tau} = d'$, $\tau = 1, \ldots, \infty$.

$$A_t^* = b \cdot d' / [1 - \lambda/(1+r)] \tag{23}$$

This specific condition is practically the only one which can be found in the marketing literature [3, 16, 18, 35, 48].

(2) Multiplicative carryover and advertising response [1, 17, 22, 27, 31, 34]

$$q_t = a \, q_{t-1}^{\lambda} \, A_t^{e_A} \tag{24}$$

where e_A is the short-run advertising elasticity and A_t^* is obtained as

$$A_t^* = (e_A \; a \; q_{t-1}^{\lambda} \; m_t')^{1/(1-e_A)} \tag{25}$$

(3) Linear, time-invariant carryover and price response [42]

$$q_t = a + \lambda q_{t-1} - c \; p_t \tag{26}$$

Optimal price:
$$p_t^* = \frac{a + \lambda q_{t-1} + c(C_t' - m_t'')}{2 \; c} \tag{27}$$

(4) Multiplicative , time-invariant carryover and price response

$$q_t = a \; q_{t-1}^{\lambda} \; p_t^{e_p} \tag{28}$$

Optimal price:
$$p_t^* = e_p(C_t' - m_t'')/(1 + e_p) \tag{29}$$

Note that in this case p_t^* is obtained by applying a constant mark-up on dynamic marginal cost, "cost plus pricing" is an optimal decision rule.

(5) Linear carryover and price response, logarithmic advertising response [12, 29, 30]

$$q_t = a + \lambda q_{t-1} + b \cdot \ln A_t - c \; p_t \tag{30}$$

Optimal price:
$$p_t^* = \frac{a + \lambda q_{t-1} + b \cdot \ln A_t^* + c(C_t' - m_t'')}{2c}$$

Optimal advertising according to (22) or (23) with p_t^* inserted.

(6) Multiplicative carryover, price response and advertising response [7, 17, 18, 23]

$$q_t = a \; q_{t-1}^{\lambda} \; A_t^{e_A} \; p_t^{e_p} \tag{31}$$

Optimal marketing mix:

p_t^* according to (29)

$$A_t^* = (e_A \; a \; q_{t-1}^{\lambda} \; p_t^{*e_p} \; m_t')^{1/(1-e_A)} \tag{32}$$

Note that p_t^* is <u>independent</u> from A_t^* whereas A_t^* depends on p_t^*. Since p_t^*

depends, in turn, on the price elasticity e_A, the latter actually determines the marketing strategy. This relationship can lead to some strange results as the author has shown for a specific model [15] in [37].

Condition (32) applies analogously to all other non-price marketing variables. The relation between the optimal values of two non-price variables (Dorfman-Steiner) is simply determined by the ratio of their elasticities.

$$x^*_{i,t}/x^*_{j,t} = e_{i,t}/e_{j,t} \tag{33}$$

Thus, if anyone of the non-price variables has been determined, all others can easily be computed from (33).

It should also be noted that the equilibrium conditions usually given in the literature [18, 19, 32] have general validity only for functions (21) and (28). In all other cases discussed here the optimal value depends on the value of the state variable which only by accident is at equilibrium.

NUMERICAL OPTIMIZATION

All expressions derived in the preceding section are necessary optimality conditions. In general, they do not readily allow for a calculation of the optimal values in period t since these values typically depend on the values which the marketing variables will attain in later periods. The marketing multiplier m''_t is a function of future contribution margins and, thereby, of future prices, so that the optimum in period t cannot be determined independently from these future prices.

Typically, the maximization of (7) involves a dynamic optimization problem, the exact solution of which can only be found by means of dynamic programming, branch-and-bound-, or nonlinear programming methods. In fact, all of these methods have been applied to problems of this type [6, 8, 27, 38, 45, 47].

The outcome of such an optimization is a series of optimal values $x^*_{j,t},\ldots,x^*_{j,T}$ only the first one of which has the property to be binding in the sense that an immediate decision has to be made and becomes effective. All other values are tentative and will not become effective before period $t+\tau$, $\tau=1,\ldots,T$. Their actual realization depends on forthcoming, typically uncertain events.

This characteristic constitutes a fundamental difference between a multi-period marketing problem and, for instance, a mathematically equivalent multi-stage production problem, in which all decision variables become effective at the same time and, hence, do not depend on future uncertain events.

As far as we can see, this problem has received very little attention in the marketing literature.

Besides the general difficulties which characterize the relationship manager - quantitative marketing model the following particular issues occur in the case of long-run optimization models:

- Usually a manager will not be able to conduct a dynamic optimization himself. Unless a qualified management scientist is available such an optimization will not be run at all.
- Though managers are typically aware that present marketing actions have carry-over-effects they are almost exclusively interested in the decision to be made now. It seems hard to convince them that decisions are also to be made for the 4th, 5th or 10th period after period t and that these decisions shall influence the decision to be made now. Weinberg [47] gives a short discussion of this issue.
- According to our experience, managers do not attribute much weight to the future optimal values in the sense that they really believe in their realization; they are, on the contrary, highly aware of their uncertain nature.

In view of these limitations and the prevailing non-acceptance of complex optimization models we suggest a rigorously simplified procedure which extends an idea of Weinberg [47] who proposed for a linear function a so-called "dynamic correction factor" in order to account for effects which occur after the planning horizon. This correction factor is a special case of our value adjusted marketing multiplier.

Instead of optimizing the marketing strategy over a number of periods t,...,T we propose to optimize only the marketing action in period t and to account for cumulative effects by means of the marketing multiplier.

In addition to the dynamic sales response function the knowledge (or at least a subjective notion) of which is required for any type of rational marketing decision we need only a few more informations:
Linear carryover-function: estimates of future contribution margins
Multiplicative carryover-function: estimates of the future values of all marketing variables and current sales q_t.

According to our experience, managers often have a rather clear-cut notion of the probable magnitudes of these variables so that the required estimates should be obtainable without great difficulties. This is in particular true for established products.

After these informations and a discount rate have been supplied the multiplier can easily be computed and used to determine the optimal value(s) of the marketing variable(s). The optimal solution can be obtained either directly [e.g. conditions (22), (23), (25), (27), (29), (32)] or, in the general case, by searching values for which equations (12) and (13) hold. Normally a pocket calculator is sufficient to carry out those calculations.

In certain cases, e.g. linear carryover and assumed constant contribution margins the procedure can be further simplified by preparing a table of

marketing multipliers for a variety of possible carryover-coefficients.

In order to discuss the non-optimality associated with the proposed technique we distinguish between a theoretical or ex-ante-error and two actual or ex-post-errors. This issue is clarified in figure 1.

INSERT FIGURE 1 HERE

The heuristic is certainly likely to involve an ex-ante-error. It is, however, hard to believe that, on the average, the ex-post-error of the heuristic is likely to be greater than the ex-post-error of the exact optimization or, in other words, that best managerial estimates should yield a less accurate prediction of future prices, costs etc. than an optimization program. Due to the fact that, in a real world market, most of the factors influencing the future actions are not under control of the decision maker rather the reverse seems true.

It is also not difficult to show that - within realistic magnitudes of m''_t - the optimal values of the marketing variables are not very sensitive to moderate changes in the estimate of future contribution margins. As we shall see subsequently the typical magnitude of m''_t seems to be about 20% of marginal cost or less. A 25%-change in the estimation of $d'_{t+\tau}$ then leads to a change in the optimal price or advertising value of about 5% or less (depending on function type and elasticities).

Another question which arises is related to the appropriate number T of periods to be included in the calculation of the marketing multiplier which normally cannot be calculated for $T=\infty$ (see table 2). A simple rule of thumb can be provided for a linear model with assumed constant contribution margins. In this model $\alpha\%$ of the total cumulative value effects occur within

$$\ln(1 - \alpha/100)/\ln[\lambda/(1+r)] \qquad (34)$$

data interval units, (34) is the discount-adjusted form of the term derived in [10, 16, 28].

Thus, if the multiplier shall account for e.g. at least 90% of the total effect and $\lambda = .55$ (Clarke [10]: mean of 59 studies $\bar{\lambda} = .534$, Simon [39]: mean of 198 products: $\bar{\lambda} = .595$) and the discount rate is 10% the next integer greater than $[\ln .1 / \ln(.55/1.1)] = 3.01$ or $T=4$ data interval units is appropriate.

In practical applications T need hardly be greater than 6 or 7 which , in turn, easens the calculation of the multiplier. The rule for T holds in the described exactness only for the linear model, but it is also a good approximation for the multiplicative model.

APPLICATION

In this section, we apply the conditions derived earlier to a number of empirically tested models in order to determine dynamic advertising and/or price optima. It is particularly interesting to compare these optima with their static counterparts in order to get a notion of the magnitude of the dynamic effects. Since it is not our primary objective to discuss the issue of competitive reaction and usually no information on this issue is supplied in the published articles we subsequently assume that the competitive marketing activities are given.

We would like to emphasize that the reaction function is much less a mathematical problem than a problem of empirical verification and foundation, for some of the difficulties see e.g. [18, 19], and it can hardly be doubted that competitive reaction functions belong to the empirically worst founded issues in marketing. Our non-reaction hypothesis limits, of course, the comparability of "optimal" and actual values of the marketing variables.

If not mentioned otherwise we apply a discount rate of 10% in the subsequent cases.

Model 1: Linear, time-invariant carryover, logarithmic advertising response, J. L. Simon [36]

The market share function is

$$m_t = \lambda\, m_{t-1} + b' \log A_t \tag{35}$$

In order to obtain the sales volume we multiply (35) by the mean market sales $\bar{R}$ [36, p. 308] and use the transformation $\log A_t = \ln A_t / \ln 10$ so that we obtain

$$q_t = \lambda\, q_{t-1} + b \cdot \ln A_t \quad \text{where } b = b' \cdot \bar{R} / \ln 10 \tag{36}$$

which is identical with (21) for a=0. Since contribution margins d' are assumed to be constant condition (23) applies and A_t^* can easily be computed. The results for the 14 liquor brands for which Simon [36] has obtained reasonable parameter estimations are given in table 3.

INSERT TABLE 3 HERE

Under the assumptions of this case, A* represents the exact optimal solution; note, in particular, that A* does not depend on the state of the system. On the average, the optimal dynamic advertising expenditures are 4.4 times the optimal static values. Condition (23) gives a good illustration of the point made by Simon [36, p.309] that A* is equally sensitive to contribution margin, carryover-effect, and advertising response. Thus, the fact that some of the A* show substantial deviations form the actual values reported in [36] may be attributable to errors in the estimation of any of these factors as well as to inadequacies in the reaction hypothesis or in actual advertising spending. The deviations between optimal and respective actual values are of both positive and negative sign and show no systematic pattern.

Model 2: Linear, time-varying carryover, logistic advertising response, ADBUDG-model, Little [20]

In our usual notation the sales function is

$$q_t = \lambda_t q_{t-1} + b_t\, x^* \left[\frac{A_t^{\alpha}}{\delta + A_t^{\alpha}} \right] \tag{37}$$

where λ_t and b_t depend on non-advertising effects and on a seasonal index, x^* is the product class sales rate and α and δ are constants. Advertising is expressed as a multiple of maintenance advertising $\bar{A}$ = 486,000 \$.

The necessary condition for optimal dynamic advertising is obtained as

$$m_t' \frac{b_t\, x^*\, \alpha\ \ \delta\, A_t^{\alpha-1}}{(\delta + A_t)^2} - \bar{A} = 0 \tag{38}$$

For the given parameter values $x^* = 2.9 \cdot 10^8$, $\alpha = 2.36$, $\delta = 4.33$ (for details see Little [21] and Weinberg [47]) and a discount factor of .93 which makes our results comparable to Weinberg's, the optimal values given in table 4 are obtained.

INSERT TABLE 4 HERE

The value multiplier m_t' (including period t) has been calculated for T = 200. As in model 1, the A*'s represent mathematically exact optima since future contribution margins are considered as given. The results of Weinberg [47] who obviously applied a unified correction factor for all periods come very close to these exact solutions.

This model is particulary interesting in that the optimal static advertising expenditure would be zero. The advertising response in period t is too small to recover the outlays in this period. Under dynamic aspects, however, advertising turns out to be highly profitable due to the high carryover-coefficient (λ_t in table 4).

For period 1, the discrepancy between dynamic and static objective function values is clarified in figure 2.

INSERT FIGURE 2 HERE

Model 3: Linear, time-invariant carryover, linear price response, Telser [42]

The price response function has the form

$$q_t = a + \lambda q_{t-1} + c\, p_t \qquad (39)$$

where q_t stands for market share and p_t denotes a relative price (ratio of the product's price and the average price of competing products). Thus, the optimization outcome is also a relative price of the same dimension. Since Telser [42] does not give marginal cost figures we are obliged to make an assumption on contribution margins, we assume that the margin accounts for 30% of the actual average price. We also assume that q_{t-1} is equal to the actual average market share reported in [42]. We take advantage of condition (27) in order to compute p_t^*, the resulting optimal dynamic and optimal static prices for Telser's 4 instant coffee brands are given in table 5.

INSERT TABLE 5 HERE

On the average, the optimal dynamic price turns out to be 21.8% lower than its static counterpart. Due to the differences in the individual value multipliers,however,the variations across brands are considerable.

Model 4: Linear, time-varying carryover, nonlinear price response, H. Simon [38]

The dynamic price response function has the form

$$q_t = a + \lambda \cdot \alpha^{\bar{t}-t} q_{t-1} + c_1 \cdot \sinh(c_2 \cdot \Delta p_t) \qquad (40)$$

where a, λ, $0<\alpha<1$, c_1, and c_2 are parameters; $\bar{t}$ denotes the product's

period of market introduction, and Δp_t is the price differential between the product under consideration and competing products.

The optimality condition is

$$p_t^* = e_{p,t}(C_t' - m_t'')/(1 + e_{p,t}) \tag{41}$$

where according to table 2 and (6) (for T=5)

$$m_t'' = \sum_{\tau=1}^{5} d_{t+\tau}' \left(\frac{\lambda}{1+r}\right)^{\tau} \alpha^{\tau t+\tau(\tau-1)/2} \tag{42}$$

We apply condition (41) to a pharmaceutical product (product 4.2 in [38]) with the following numerical values: a=936, λ=.756, α=.96, c_1=45.7, c_2=6.71, C_t' = .20, and estimated future contribution margins of $d_{t+\tau}'$ = .45. In addition to the usual dynamic and static values we also give the results of an exact branch-and-bound-optimization in table 6.

INSERT TABLE 6 HERE

The comparison of columns (2) and (3) in table 6 reveals a considerable conformity between branch-and-bound- and heuristic results in both magnitudes and changes in prices over time. Whereas the conformity of the magnitudes is mainly due to the choice of $d_{t+\tau}'$ similar trajectory types would be produced within a wide range of reasonable contribution margins as one can easily infer from (41) and (42). This latter finding seems particularly important since the determination of the appropriate trajectory type (e.g. penetration or skimming strategy) has to be considered as the crucial issue in strategic pricing. This example and a few other examples not reported here indicate that the simple multiplier method performs very well with respect to this issue.

The comparison of optimal dynamic and optimal static prices (column 4 in table 6) again elucidates our analytical result that the former are smaller

than the latter, the average deviation being 6.5% in this case. Due to the influence of the "obsolescence term" $\alpha^{\bar{t}-t}$ the differences between the two prices decrease over time.

Though very little empirical evidence is available to date, the value multiplier m''_t seems to be significantly different for various product groups. For model (40) the author found a mean value of 10.3% for 13 detergents and of 40.3 % for 22 pharmaceuticals investigated in [38], the percentage is related to marginal cost with an assumed contribution margin of 30%. The values apply to the period of introduction, they are lower in later periods of the product's life. Thus, due to the considerably higher carryover-coefficient (.78 versus .32, see [39]) long-run considerations should play a more important role for pharmaceuticals than for detergents.

Model 5: Multiplicative, time-invariant carryover, price response, and advertising response, Lambin [18], Moriarty [23]

The response function has the form

$$q_t = a\, q_{t-1}^{\lambda}\, p_t^{e_p}\, A_t^{e_A} \tag{43}$$

Since none of the empirical models reported in the literature provides sufficient information for optimization purposes we use the following numerical example for demonstration: $a=10$, $q_{t-1}=100$, $\lambda=.55$, $e_p=-2$, $e_A=.10$. Price is expressed as a multiple of marginal cost $C'_t=1$ and the future values of prices and advertising are estimated to be $p_{t+\tau}=1.3 (\rightarrow d'_{t+\tau}=.3)$ and $A_t=5$ respectively.

In order to be able to compute m''_t we make the simplifying assumption that, in t, $q_t=q_{t-1}$ and according to (6) and table 2 we obtain $m''_t = .24$ which means that the value multiplier reduces (static) marginal cost by 24%. From (29) we obtain the optimal dynamic price as $p^*_t = 1.52$, its static counterpart

being $p_t^{stat} = 2$. Inserting p_t^* into (32) yields the optimal advertising expenditure as

$$A_t^* = (.1 \cdot 10 \cdot 100^{.55} \cdot 1.52^{-2} \cdot .76)^{1.11} = 4.84$$

The respective static optimum is $A_t^{stat} = 3.57$.

The former results of the optimal dynamic price being lower and the optimal dynamic advertising expenditure being higher than the respective static values are confirmed.

Inserting the optimal values into the Dorfman-Steiner-Condition (16) we recognize the analytical result that the sales-advertising-ratio is the same under dynamic and static conditions though the magnitudes of the respective dynamic and static marketing variables differ by 24% for the price and 35.5% for advertising.

$$\frac{p_t\, q_t}{A_t^*} = \frac{p_t^{stat} q_t^{stat}}{A_t^{stat}} = -\frac{e_p}{e_A}$$

$$\frac{96.8}{4.84} = \frac{71.4}{3.57} = -\frac{-2}{.1} = 20$$

In both cases, the optimal advertising expenditures account for 5% of sales.

SUMMARY

A simple heuristic method for the determination of dynamically optimal levels of marketing actions for models with separable carryover- and response effects is proposed. The core of the method is a marketing multiplier which measures the future effects of present marketing activities. Marginal cost is diminished by this multiplier and approximately optimal values of the marketing variables can be computed without applying complex optimization techniques.

Simple optimality conditions reveal very clearly the relation between dynamically and statically optimal levels of marketing activities. A dynamic version of the Dorfman-Steiner-Theorem shows that, for the class of models under consideration, the optimal sales-promotion-ratio is the same under static and dynamic conditions though the magnitudes of the variables are typically different.

Applications to a few empirical models indicate that the dynamic impact on the optimal levels of prices and advertising expenditures is,almost generally, by far too large to be neglected. The method seems to produce accurate trajectories over time.

Though the number of practical applications and experience is yet very limited managers seem to find the method intuitively appealing. They appreciate in particular its simplicity which enables them to make the necessary calculations themselves and gives easy way to sensitivity analyses.

TABLE 1: SYNOPSIS OF EMPIRICAL STUDIES OF CARRYOVER-MODELS

Carryover-Coefficient \ Function Type	Linear	Multiplicative
Constant	18 Studies	19 Studies
Time-varying	Beckwith [2] Winer [49] Simon [38]	Parsons [27]
Policy-Varying	Vidale-Wolfe [46] Little [20] Naert-Bultez [24]	Wittink [50]

TABLE 2: MARKETING MULTIPLIER m_t IN DIFFERENT EMPIRICAL MODELS

Study	Carryover		Marketing Multiplier m_t	Remarks
	Type	Function		
18 Studies	linear, constant	$\lambda\, q_{t-1}$	$\sum_{\tau=0}^{\infty} \lambda^{\tau} = \frac{1}{1-\lambda}$	$0 < \lambda < 1$
19 Studies	multiplicative, constant	$a_t\, q_{t-1}^{\lambda}$	$1 + \sum_{\tau=1}^{\infty} \lambda^{\tau}\, q_t^{\lambda^{\tau}-1} \left[\prod_{\tau_1=1}^{\tau} a_{t+\tau_1}^{\lambda^{\tau-\tau_1}}\right]$	$a_t = a_0 \prod_{j=1}^{n} x_{j,t}^{e_j}$
Beckwith [2]	linear, time-varying	$(\lambda_1+\lambda_2 t)q_{t-1}$	$1 + \sum_{\tau=1}^{\infty} \prod_{\tau_1=1}^{\tau} [\lambda_1 + \lambda_2(t+\tau_1)]$	
Parsons [27]	multiplic., time-varying	$a_t q_{t-1}^{\lambda_1(1-e^{\lambda_2 t}) + \lambda_3}$	$1 + \sum_{\tau=1}^{\infty} \lambda_{\tau}\, q_t^{\lambda_{\tau}-1} \left[\prod_{\tau_1=1}^{\tau} a_{t+\tau_1}^{\lambda_{\tau}/\lambda_{\tau_1}}\right]$	$\lambda_{\tau} = \prod_{\tau_1=1}^{\tau} [\lambda_1(1-e^{-\lambda_2(t+\tau_1)})+\lambda_3]$
Little [20] Winer [49]	linear, time-varying	$\lambda_t\, q_{t-1}$	$1 + \sum_{\tau=1}^{\infty} \prod_{\tau_1=1}^{\tau} \lambda_{\tau_1}$	
Simon [38]	linear, time-varying	$\lambda\, r^t\, q_{t-1}$	$\sum_{\tau=0}^{\infty} \lambda^{\tau}\, r^{\tau t+\tau(\tau-1)/2}$	$0 < r < 1$

TABLE 3: OPTIMAL ADVERTISING LEVELS FOR DIFFERENT LIQUOR BRANDS

Brand	carryover λ	multiplier m'_t	adv. response b	contribution d'	optimal advertising [$1000]	
					dynamic A^*_t	static A^{stat}_t
Guckenheimer	.87	4.78	8.68	1.67	69	14
Carstairs	.67	2.56	17.67	2.59	117	46
Four Roses	.44	1.67	32.82	7.65	419	251
Kessler	.98	9.17	13.89	2.81	358	39
Seagrams 7	.75	3.14	98.47	6.46	1997	636
Corby's	.74	3.06	45.44	3.20	443	145
Imperial	.88	5.00	20.20	3.25	328	66
Old Thompson	.83	4.07	3.41	3.32	46	11
Ancient Age	.67	2.56	16.11	8.15	336	131
Echo Springs	.84	4.23	11.87	3.65	183	43
Kentucky Gentleman	.89	5.24	17.81	3.40	317	61
Bourbon Supreme	1.00	11.00	55.97	2.75	1693	154
Gordon's	.73	2.97	4.96	.27	4	1.3
Gilbey's Gin	.61	2.24	22.47	1.38	69	31

TABLE 4: OPTIMAL ADVERTISING ADBUDG-MODEL [$ 1000]

period t	carryover λ_t	adv.response b_t	multiplier m'_t	optimal advertising A^*_t	Weinberg-result [47]
1	.934	.0045	6.18	2069	2078
2	1.07	.005	5.52	2063	2084
3	.933	.005	5.58	2071	2079
4	.873	.0046	6.04	2069	2075

TABLE 5: OPTIMAL RELATIVE PRICES FOR 4 INSTANT COFFEE BRANDS

Brand	contribution d'	multiplier m''_t	optimal prices dynamic p^*_t	optimal prices static p^{stat}_t	deviation [%]
A	.355	.388	1.77	1.99	12.4
B	.216	.453	.76	.98	28.9
C	.388	.133	1.01	1.07	6.5
D	.288	.747	.94	1.31	39.3

TABLE 6: OPTIMAL PRICES FOR A PHARMACEUTICAL PRODUCT IN DIFFERENT PERIODS

Period (1)	optimal prices			actual competitive price* (5)
	branch-and-bound (2)	heuristic (3)	static (4)	
1	.50	.54	.61	.48
2	.60	.60	.66	.49
3	.62	.63	.68	.49
4	.64	.64	.68	.49
5	.67	.67	.71	.52
6	.70	.69	.72	.54
7	.70	.69	.72	.54
8	.70	.71	.74	.55
9	.70	.71	.74	.55
10	.75	.71	.74	.55

* market share weighted average of competing products

FIGURE 1: EX-ANTE- AND EX-POST-ERRORS IN A DYNAMIC DECISION MODEL

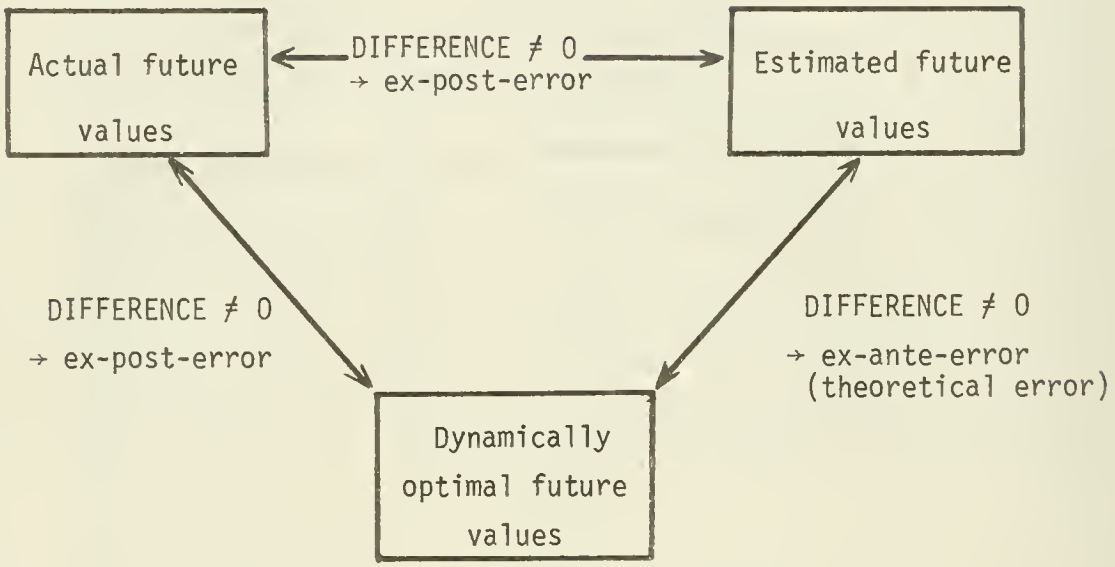
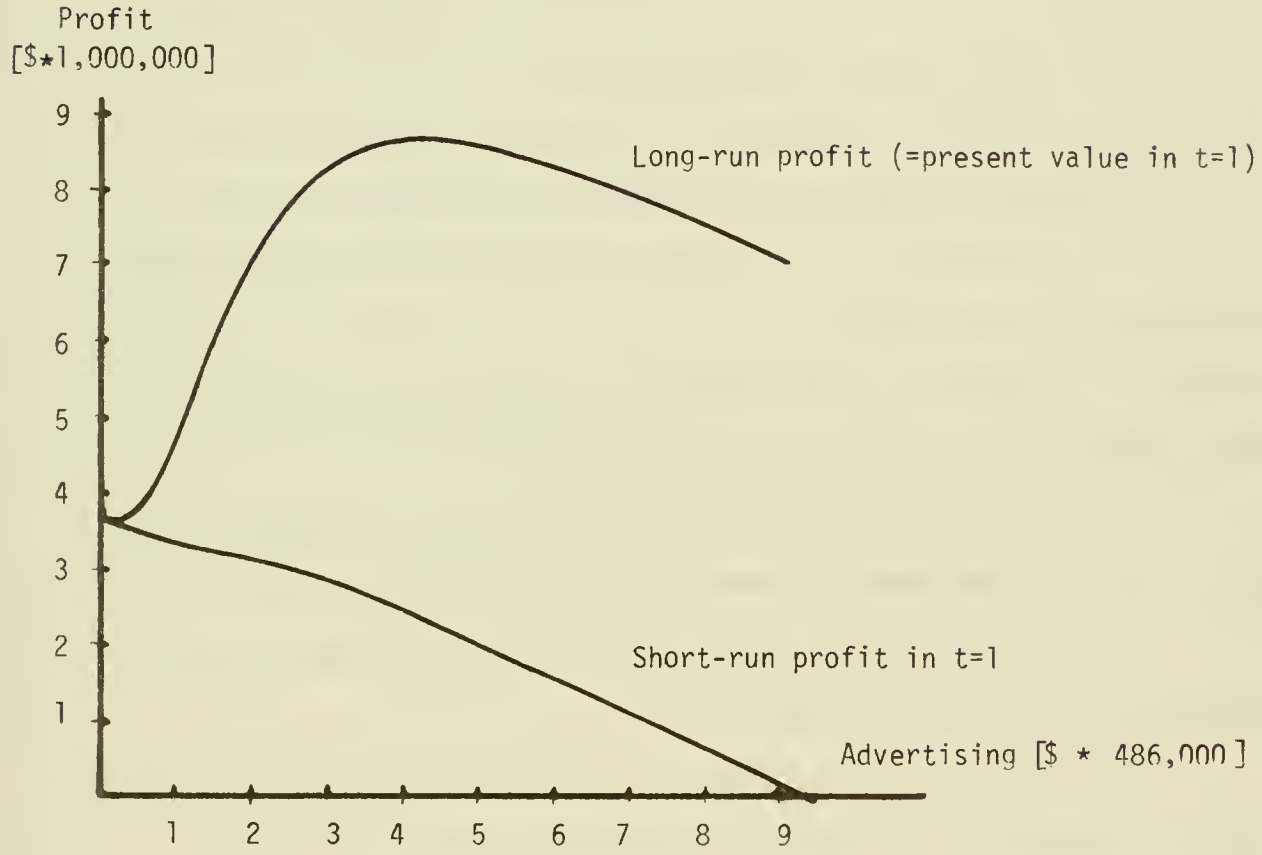


FIGURE 2: LONG-RUN PROFIT AND SHORT-RUN PROFIT AS FUNCTIONS OF ADVERTISING IN THE ADBUDG-MODEL



REFERENCES

1. BASS, F.M. AND PARSONS, L.J., "A Simultaneous Equation Regression Analysis of Sales and Advertising", Applied Economics 1(1969), 103-124.
2. BECKWITH, N. E., "Regression Estimation of the Time-Varying Effectiveness of Advertising", Working Paper, Columbia University 1972.
3. _____, "Multivariate Analysis of Sales Responses of Competing Brands to Advertising", Journal of Marketing Research 9(1972), 168-176.
4. BENSOUSSAN, A., BULTEZ, A. AND NAERT, PH., "Towards a Dynamic Theory of the Multiproduct Firm, A Generalization of the Nerlove-Arrow Optimality Conditions", Working-Paper 73-35, European Institute for Advanced Studies in Management, Brussels 1973.
5. BENSOUSSAN, A., HURST, G.E. AND NAESLUND, B., Management Applications of Modern Control Theory, Amsterdam: North Holland 1974.
6. BULTEZ, A., La firme en concurrence sur des marchés interdependents, Unpublished doctoral dissertation, Universite Catholique de Louvain 1975
7. _____, "A Nonlinear Dynamic Model of Product Line Interdepencies", Working Paper 75-6, EIASM Brussels 1975.
8. _____ AND NAERT, PH., "Does Lag Structure Really Matter in Optimizing Advertising Spending?", forthcoming in Management Science.
9. BUZZELL, R. D., "Predicting Short-Term Changes in Market Share as a Function of Advertising Strategy", Journal of Marketing Research 1(1964, August), 27-31.
10. CLARKE, D.G., "Econometric Measurement of the Duration of Advertising Effect on Sales", Journal of Marketing Research 13(1976), 345-357.
11. DHALLA, N.K., "Assessing the Long-Term Value of Advertising", Harvard Business Review 56(1978, Jan.-Febr.), 87-95.

12. HOUSTON, F.S. AND WEISS, D.L., "An Analysis of Competitive Marketing Behavior", Journal of Marketing Research 11(1974), 151-155.

13. IRELAND, N.J. AND JONES, H.G., "Optimality in Advertising: A Control Theory Approach", Proceedings of the IFORS/IFAC International Conference, Coventry (UK) 1973, 186-199.

14. JACQUEMIN, A.P., "The Dynamic Analysis of Advertising Policy", Working Paper 72-20, EIASM, Brussels 1972.

15. KOTLER, PH., "Competitive Strategies for New Product Marketing over the Life Cycle", Management Science 12(1965, No. 4), B-104 - 119.

16. _____, Marketing Decision Making: A Model Building Approach, New York: Holt, Rinehart and Winston 1971.

17. LAMBIN, J.J., "Measuring the Profitability of Advertising: An Empirical Study", Journal of Industrial Economics 17(1969), 468-484.

18. _____, Advertising, Competition and Market Conduct in Oligopoly over Time, Amsterdam: North Holland 1976.

19. _____, NAERT, PH. AND BULTEZ, A., "Optimal Market Behavior in Oligopoly", European Economic Review 6(1975), 105-128.

20. LITTLE, J.D.C., "Models and Managers: The Concept of a Decision Calculus", Management Science 16(1970), B-466 - 485.

21. _____, "Decision Support for Marketing Management", Working Paper 78-A.P. Sloan School of Management, MIT, Cambridge (Mass) 1978.

22. MONTGOMERY, D.B. AND SILK, A.J., "Measuring Dynamic Effects of Market CommunicationsExpenditures", Management Science 18(1972), B-485-501.

23. MORIARTY, M., "Cross-Sectional, Time Series Issues in the Analysis of Marketing Decision Variables", Journal of Marketing Research 12(1975), 142-150.

24. NAERT, PH. AND BULTEZ, A., "A Model of a Distribution Network Aggregate Performance", Management Science 21(1975), 1102-1112.

25. NERLOVE, M. AND ARROW, K.J., "Optimal Advertising Policy under Dynamic Conditions", Economica 29(1962), 129-142.

26. PALDA, K. S., The Measurement of Cumulative Advertising Effects, Englewood Cliffs: Prentice Hall 1964.

27. PARSONS, L.J., "The Product Life Cycle and Time-Varying Advertising Elasticities", Journal of Marketing Research 12(1975), 476-480.

28. _____ AND SCHULTZ, R.L., Marketing Models and Econometric Research, New York: American Elsevier 1976.

29. PELES, Y., "Rates of Amortization of Advertising Expenditures", Journal of Political Economy 79(1971), 1032-1058.

30. PRASAD, K.K. AND RING, W.L., "Measuring Sales Effects of Some Marketing Mix Variables and their Interactions", Journal of Marketing Research 13(1976), 391-396.

31. SAMUELS, J.M., "The Effect of Advertising on Sales and Brand Shares", European Journal of Marketing 4(1970/71), 187-207.

32. SCHMALENSEE, R., The Economics of Advertising, Amsterdam: North Holland 1972.

33. SETHI, S.P, "Optimal Control of the Vidale-Wolfe Advertising Model", Operations Research 21(1973), 998-1013.

34. SEXTON, D.E., "Estimating Marketing Policy Effects on Sales of Frequently Purchased Products", Journal of Marketing Research 7(1970), 338-347.

35. SIMON, J.L., "A Simple Model for Determining Advertising Appropriations", Journal of Marketing Research 2(1965), 285-292.

36. _____, "The Effect of Advertising on Liquor Brand Sales", Journal of Marketing Research 6(1969), 301-313.

37. SIMON, H., "An Analytical Investigation of Kotler's Competitive Simulation Model", Management Science 24(1978, No. 14), 1462-1473.

38. _____, "Dynamics of Price Elasticity and the Product Life Cycle", Working Paper 1035-78, A.P. Sloan School of Management, MIT, Cambridge (Mass.) 1978.

39. _____, "Dynamische Erklaerungen des Nachfragerverhaltens aus Carryover-Effekt und Responsefunktion", in: MEFFERT, H., FRETER, H. AND STEFFENHAGEN, H. (eds.), Konsumentenverhalten und Information, Wiesbaden: Gabler 1979.

40. SPREMANN, K., "Optimale Preispolitik bei dynamischen deterministischen Absatzmodellen", Zeitschrift fuer Nationalökonomie 35(1975), 63-76.

41. _____ AND OPTIZ, O., "Optimale Steuerung von Kaufverhaltensprozessen", in: TOPRITZHOFER, E. (ed.), Marketing - Neue Ergebnisse aus Forschung und Praxis, Wiesbaden: Gabler 1978.

42. TELSER, L.G., "The Demand for Branded Goods as Estimated from Consumer Panel Data", The Review of Econoics and Statistics 44(1962), 300-324.

43. TSURUMI, H. AND TSURUMI, Y., "Simultaneous Determination of Market Share and Advertising Expenditure under Dynamic Conditions: The Case of a Firm within the Japanese Pharmaceutical Industry", The Economics Studies Quarterly 22(1971), 1-23.

44. TURNER, R.E. AND NEUMANN, C.P., "Dynamic Advertising Strategy: A Managerial Approach", Journal of Business Administration 7(1975, Spring), 1-21.

45. URBAN, G.L., "An On-Line Technique for Estimating and Analyzing Complex Models", in: Changing Marketing Systems, Proceedings American Marketing Association, Chicago 1971.

46. VIDALE, M.L. AND WOLFE, H.B., "An Operations Research Study of Sales Response to Advertising", Operations Research 5(1957)3, 370-381.

47. WEINBERG, CH., "Dynamic Correction in Marketing Planning Models", Management Science 22(1976), 677-687.

48. WELAM, P., "Carryover and Optimal Advertising - II", Journal of Advertising Research 15 (1975, June), 43-45.

49. WINER, R.S., "A Time-Varying Parameter View of the Sales-Advertising Relationship", Educators' Proceedings, American Marketing Association, Chicago 1976, 461-465.

50. WITTINK, D.R., "Exploring Territorial Differences in the Relationship between Marketing Variables", Journal of Marketing Research 14(1977),145-155.

BASEMENT

JAN 11

FEB

MAR 15 1985

OCT 7 1986

JUN

JUN

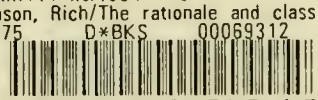
ACME
BOOKBINDING CO., INC.
SEP 6 1983
100 CAMBRIDGE STREET
CHARLESTOWN, MASS.

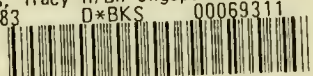
HD28.M414 no.1049- 79
Little, John D/The implications of ext
736027 D*BKS 00066620

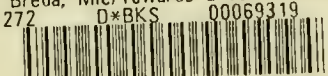
3 9080 001 209 334

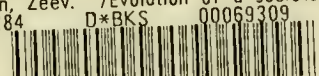
HD28.M414 no.1050- 79
Simon, Hermann/Marketing multiplier an
736280 D*BKS 00069321

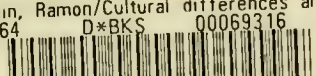
3 9080 001 240 776

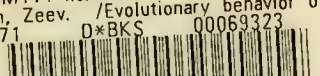
HD28.M414 no.1051- 79
Robinson, Rich/The rationale and class
736275 D*BKS 00069312

3 9080 001 240 602

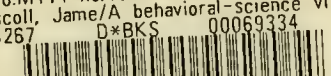
HD28.M414 no.1052- 79
Lewis, Tracy R/On oligopolistic market
736283 D*BKS 00069311

3 9080 001 240 586

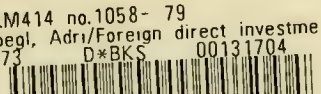
HD28.M414 no.1053- 79
Van Breda, Mic/Towards a definition of
736272 D*BKS 00069319

3 9080 001 240 735

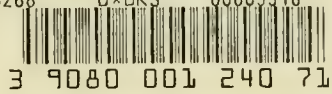
HD28.M414 no.1054- 79
Bonen, Zeev. /Evolution of a sociotec
736284 D*BKS 00069309

3 9080 001 240 545

HD28.M414 no.1055- 79
Barquin, Ramon/Cultural differences an
736264 D*BKS 00069316

3 9080 001 240 685

HD28.M414 no.1056- 79
Bonen, Zeev. /Evolutionary behavior o
736271 D*BKS 00069323

3 9080 001 240 818

HD28.M414 no.1056- 79a
Driscoll, Jame/A behavioral-science vi
736267 D*BKS 00069334

3 9080 001 240 982

HD28.M414 no.1058- 79
Tschoegl, Adri/Foreign direct investme
830173 D*BKS 00131704

3 9080 001 984 639

HD28.M414 no.1060- 79
Wallace, Phyll/Affirmative action and
736268 D*BKS 00069318

3 9080 001 240 719